Anatol Vieru's Periodic Sequences

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Abstract:
In the context of 1960's musical serialism, the romanian composer Anatol Vieru explores a composing technique based on periodic sequences with values in $\mathbb{Z}/12\mathbb{Z}$: starting from one of his choice, let's say f, he generates several periodic sequences from it by applying either the differential operator Δ or the integral operator Σ. $\Sigma^n f$ are thought as additive layers for the initial theme f, and Vieru decodes a musical aspect from each layer with a suitable dictionary. With this technique, Vieru composes several pieces. Moreover, he notices that when applying Σ the period generally increases, and (in the cases he studied) all the periods are always powers of 2. Furthermore, manipulating the sequence $(2, 1)$ on $\mathbb{Z}/12\mathbb{Z}$, he gets the sequence $(2, 1, 2, 4, 8, 1, 8, 4)$ and he notices that in the coefficients of its primitives the values 4 and 8 proliferate. Such questions motivated more recent investigations based on a computational approach, which confirmed Vieru's observations.
In this seminar we will provide the solutions to these questions in a purely algebraic way. To do so, we will firstly recall the link between periodic sequences with coefficients in $\mathbb{Z}/m\mathbb{Z}$ (that we will call P_m) and the binomial coefficients modulo m. Then, we will decompose the $\mathbb{Z}/m\mathbb{Z}$-module P_m not only in primes, but also into nilpotent and idempotent part. To the aim of determining the period of the primitive of a generic sequence, we will reduce both nilpotent and idempotent sequences to the study of constant sequences. Lastly, we will manage to express the coefficients of the primitives of Vieru's sequence in a recursive way.

References:
[1] L. Fiorot, A. Tonolo and R. Gilblas, *The Mystery of Anatol Vieru's Periodic Sequences Unveiled*, Mathematics and Computation in Music, (2022), pp. 376-382.
[2] M. Andreatta, D.T. Vusa and C. Agon, *On some theoretical and computational aspects of Anatol Vieru's periodic sequences*, Soft Computing 8, no.9, (2004), pp. 588-596.